The Bitcoiner's Dilemma

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I'd like to present a simplified analysis of the bidding that took place for the USMS's bitcoin this past Friday. We model the scenario as a two player zero-sum game between me (I would really like to own those coins) and a single other bidder. Both players may choose from the following set of moves

$$S = \{\text{ABOVE MARKET},\ \text{AT MARKET},\ \text{BELOW MARKET}\}.$$

We will assume that if both players submit a bid at market price, a coin is flipped to decide the winner. The situation becomes complicated when the players submit matching bids of either both above or both below market price. One could even imagine a matrix game within a matrix game where once we know both bids are in the same range the players enter a new game where the moves become 5%, 10%, or 15% above/below market price.

For simplicity's sake, if both players submit matching bids, a coin is flipped to decide the winner. The matrix game is then

$$G = \begin{array}{c} \\ AM \\ @M \\ BM \end{array} \begin{array}{c} \begin{array}{ccc} AM & @M & BM \end{array} \\ \begin{pmatrix} 0.5 & 1 & 1 \\ 0 & 0.5 & 1.5 \\ 0 & 0 & 0.5 \end{pmatrix} \end{array}$$

where the payoffs are to me, "player one". In this case my moves correspond to selecting rows so my maximin strategy is $x = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t$. There are a few ways to arrive at this answer and we will begin with the simplest before looking briefly at the math behind it.

If we don't want to dirty our hands with the computation once we've set up our game we can use a solver readily available from UCLA. Just plug in the matrix and let it spit out the answer.

However, if we are so inclined, we can find the optimal strategy ourselves by solving a linear program. Let $x' = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^t$ be the probability vector that describes my bidding strategy. That is, x_1 is the probability that I bid *above* the market price, x_2 is the probability that I bid *at* the market price, and x_3 is the probability that I bid *below* the

market price. To find our optimal strategy, we must solve the following LP

$$\begin{aligned} \text{Maximize} \quad & \min\{0.5x_1,\ x_1+0.5x_2,\ x_1+1.5x_2+0.5x_3\} \\ \text{Subject to} \quad & x_1+x_2+x_3=1 \\ & x_1,x_2,x_3 \geq 0 \end{aligned}$$

We wish to maximize our minimum payoff, which depends on the move our opponent ("player two") selects. The functions whose minimum we wish to maximize correspond to player two's decision to bid above, at, or below market price, respectively. However, the min function is not linear so we must transform the LP to an equivalent one that a linear solver can handle

$$\begin{aligned} \text{Maximize} \quad & x_4 \\ \text{Subject to} \quad & 0.5x_1 \geq x_4 \\ & x_1+0.5x_2 \geq x_4 \\ & x_1+1.5x_2+0.5x_3 \geq x_4 \\ & x_1+x_2+x_3=1 \\ & x_1,x_2,x_3 \geq 0 \end{aligned}$$

Since there are no real constraints on x_4, maximizing it "pushes" its value up until x_4 is exactly equal to $\min\{0.5x_1,\ x_1+0.5x_2,\ x_1+1.5x_2+0.5x_3\}$. Hence we have achieved our goal of maximizing our minimum payoff.

The following R code solves the LP for us

```
> library(lpSolve)
> obj <- c(0, 0, 0, 1)
> con <- matrix(c(0.5, 0, 0, -1,
+                  1, 0.5, 0, -1,
+                  1, 1.5, 0.5, -1,
+                  1, 1, 1, 0), byrow = T, nrow = 4)
> rhs <- c(0, 0, 0, 1)
> dir <- c(">=", ">=", ">=", "=")
> lp("max",obj, con, dir, rhs)

Success: the objective function is 0.5

> lp("max",obj, con, dir, rhs)$solution

[1] 1.0 0.0 0.0 0.5
```

Thus the optimal values are $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0.5$ and our optimal strategy is to always (with probability 1) bid above the market price.

An even easier way to arrive at this solution is by using dominance to eliminate possible moves. We can throw out move three because I am guaranteed to always do at least as good or better if I play move one than if I were to play move three. Hence, the game I see may be simplified to

$$G' = \begin{array}{c c} & \begin{array}{ccc} AM & @M & BM \end{array} \\ \begin{array}{c} AM \\ @M \end{array} & \begin{pmatrix} 0.5 & 1 & 1 \\ 0 & 1 & 1.5 \end{pmatrix} \end{array}$$

However, from player two's standpoint move one dominates moves two and three, so the payoff matrix becomes

$$G'' = \begin{array}{c c} & AM \\ \begin{array}{c} AM \\ @M \end{array} & \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} \end{array}$$

And now move one for player one dominates move two, so we are left with

$$G'' = \begin{array}{c c} & AM \\ AM & \begin{pmatrix} 0.5 \end{pmatrix} \end{array}$$

Thus the optimal strategy for both players is $x = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t$. Granted, this simple model assumes that winning the auction, not winning the most coins at the lowest cost, is the optimal approach. Still, it is a nice exercise in linear programming and exposes the reader to a few concepts from game theory.

The reader is encouraged to solve the LP a third way: graphically. Start by plotting $0.5x_1,\ x_1 + 0.5x_2$, and $x_1 + 1.5x_2 + 0.5x_3$ against one another and go from there.